ON THE EXPERIMENTAL DETERMINATION OF THERMAL FLUXES IN THE WALLS OF MOLDS USED FOR THE CONTINUOUS CASTING OF METALS

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The equation of the temperature field in the plane wall of a mold is derived for the case in which the surface temperature varies harmonically. A method of determining the maximum specific thermal fluxes through the mold wall under these conditions is proposed.

One of the most important characteristics of the operation of a mold used in the continuous casting of metals is the specific thermal flux transferred through the wall from the casting to the cooling medium.

The thermal flux may be determined experimentally by means of differential thermocouples with their ends placed at different depths in the mold wall. If the temperature varies linearly through the wall, the thermal flux may be simply found from

$$q = \frac{\lambda}{\Delta x} \Delta T. \tag{1}$$

Experiments show that the temperature difference between the junctions of the differential thermocouple fluctuates all the time. The characteristic curve of the thermoelectromotive force generated by the thermocouple, recorded by means of an electronic potentiometer, for the semicontinuous casting of copper in a copper mold, is shown in Fig. 1.

The change taking place in the surface temperature of the mold wall are of the same character. This effect may be explained as being due to vibrations of the casting within the mold, leading to a periodic variation of the gas space between the casting and the mold, and hence to changes in the specific thermal flux. The heat-transfer process in the mold is therefore not a steady-state process, and the temperature distribution through the wall is not linear. Any determination of the thermal flux by means of Eq. (1) may therefore lead to errors.

In this paper we shall propose a method of analyzing the experimental data when determining the thermal flux in a mold, these data being given in the form of the time dependence of the electromotive force of a differential thermocouple (Fig. 1).

First of all, let us derive an equation for the temperature field within a plane mold wall; the wall is washed on one side by a constant-temperature coolant, the temperature of the other surface varying harmonically.

In order to derive this equation, we use the differential equation of heat conduction

$$\frac{\partial T(x,\ \tau)}{\partial \tau} = a \frac{\partial^2 T(x,\ \tau)}{\partial x^2}. \tag{2}$$

The boundary condition on the surface of the wall turned toward the casting (x = 0) is:

$$T(0,\ \tau) = T_0 + T_a \sin \omega\tau. \tag{3}$$

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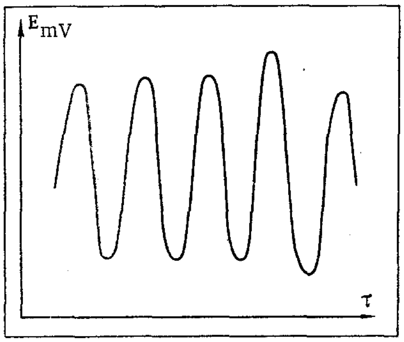

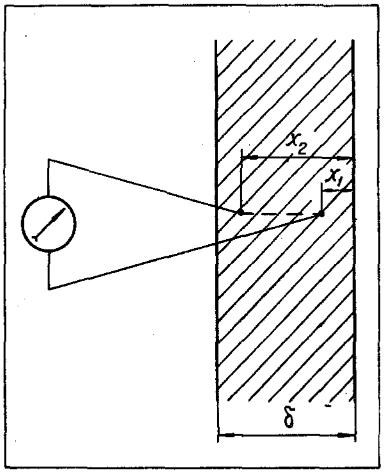

Fig. 1 Fig. 2

Fig. 1. Time variation of the electromotive force of the differential thermocouple.

Fig. 2. Arrangement of the differential thermocouple in the mold wall.

The boundary condition on the side of the cooling medium (x = δ) is

$$\alpha\,[T(\delta,\ \tau)-T_{\text{в}}]=-\lambda\left.\frac{\partial T(x,\ \tau)}{\partial x}\right|_{x=\delta}. \tag{4}$$

Let us take the initial (τ = 0) temperature distribution in the wall as linear, i.e.,

$$T(x,\ 0)=T_0-kx, \tag{5}$$

where

$$k=\frac{1}{\delta}\,\frac{T_0-T_{\text{в}}}{1+\dfrac{1}{\mathrm{Bi}}}. \tag{6}$$

The Biot criterion Bi = $\alpha\delta/\lambda$.

The solution of Eq. (2), with boundary conditions (3) and (4) and initial condition (5), was obtained by means of an integral Laplace transformation:

$$T(x,\ \tau)=T_0-kx+T_{\mathrm{a}}\sqrt{\frac{R^2(x)+S^2(x)}{V^2+U^2}}\sin[\omega\tau+\varphi(x)]$$

$$+2T_{\mathrm{a}}\sum_{n=1}^{\infty}\frac{\sin\left(\mu_n\dfrac{\delta-x}{\delta}\right)+\dfrac{\mu_n}{\mathrm{Bi}}\cos\left(\mu_n\dfrac{\delta-x}{\delta}\right)}{\mathrm{Pd}\left(\dfrac{\mu_n^4}{\mathrm{Pd}^2}+1\right)\left[\dfrac{\sin\mu_n}{\mathrm{Bi}}-\dfrac{\cos\mu_n}{\mu_n}\left(1+\dfrac{1}{\mathrm{Bi}}\right)\right]}\exp\left(-\frac{\mu_n^2 a}{\delta}\tau\right), \tag{7}$$

where the Pd criterion

$$\mathrm{Pd}=\frac{\omega\delta^2}{a};$$

$$V=\mathrm{sh}\,z\cos z+\frac{z}{\mathrm{Bi}}\,\mathrm{ch}\,z\cos z-\frac{z}{\mathrm{Bi}}\,\mathrm{sh}\,z\sin z; \tag{8}$$

$$U=\mathrm{ch}\,z\sin z+\frac{z}{\mathrm{Bi}}\,\mathrm{sh}\,z\sin z+\frac{z}{\mathrm{Bi}}\,\mathrm{ch}\,z\cos z; \tag{9}$$

$$R(x)=\mathrm{sh}\left(z\frac{\delta-x}{\delta}\right)\cos\left(z\frac{\delta-x}{\delta}\right)+\frac{z}{\mathrm{Bi}}\,\mathrm{ch}\left(z\frac{\delta-x}{\delta}\right)\cos\left(z\frac{\delta-x}{\delta}\right)-\frac{z}{\mathrm{Bi}}\,\mathrm{sh}\left(z\frac{\delta-x}{\delta}\right)\sin\left(z\frac{\delta-x}{\delta}\right); \tag{10}$$

$$S(x)=\mathrm{ch}\left(z\frac{\delta-x}{\delta}\right)\sin\left(z\frac{\delta-x}{\delta}\right)+\frac{z}{\mathrm{Bi}}\,\mathrm{sh}\left(z\frac{\delta-x}{\delta}\right)\sin\left(z\frac{\delta-x}{\delta}\right)+\frac{z}{\mathrm{Bi}}\,\mathrm{ch}\left(z\frac{\delta-x}{\delta}\right)\cos\left(z\frac{\delta-x}{\delta}\right); \tag{11}$$

$$\varphi(x) = \operatorname{arc\,tg} \frac{S(x)V - R(x)U}{R(x)V + S(x)U}; \tag{12}$$

$$z = \sqrt{\frac{\mathrm{Pd}}{2}}; \tag{13}$$

μ_n are the roots of the characteristic equation

$$\operatorname{tg} \mu = -\mu/\mathrm{Bi}. \tag{14}$$

Some time after the onset of the oscillatory mode of variation in the temperature of the mold wall a regular mode becomes established. Then Eq. (7) takes the form

$$T(x, \tau) = T_0 - kx + T_a \sqrt{\frac{R^2(x) + S^2(x)}{V^2 + U^2}} \sin[\omega\tau + \varphi(x)]. \tag{15}$$

Using (15), we obtain the time dependence of the temperature difference at points 1 and 2 (Fig. 2), i.e., the relationship indicated by the potentiometer connected to the thermoelectric pyrometer:

$$\Delta T = T(x_1, \tau) - T(x_2, \tau) = T_0 - kx_1 + T_a \sqrt{\frac{R^2(x_1) + S^2(x_1)}{V^2 + U^2}} \sin[\omega\tau + \varphi(x_1)]$$

$$- T_0 + kx_2 - T_a \sqrt{\frac{R^2(x_2) + S^2(x_2)}{V^2 + U^2}} \sin[\omega\tau + \varphi(x_2)]. \tag{16}$$

After transformation we obtain

$$\Delta T = k(x_2 - x_1) + T_a \sqrt{\frac{[R(x_1) - R(x_2)]^2 + [S(x_1) - S(x_2)]^2}{V^2 + U^2}} \sin(\omega\tau + \rho), \tag{17}$$

where

$$\rho = \operatorname{arc\,tg} \frac{V[S(x_1) - S(x_2)] - U[R(x_1) - R(x_2)]}{V[R(x_1) - R(x_2)] + U[S(x_1) - S(x_2)]}. \tag{18}$$

Using (1) and (17), we may write the equation of the specific thermal flux determined experimentally from the temperature drop at the junctions of the differential thermocouple thus:

$$q_e = \frac{\lambda}{x_2 - x_1} \Delta T = \lambda k + \frac{\lambda T_a}{x_2 - x_1} \sqrt{\frac{[R(x_1) - R(x_2)]^2 + [S(x_1) - S(x_2)]^2}{V^2 + U^2}} \sin(\omega\tau + \rho). \tag{19}$$

Now let us find the real thermal flux transferred from the casting to the mold wall:

$$q_r = -\lambda \left.\frac{\partial T(x, \tau)}{\partial x}\right|_{x=0} \tag{20}$$

Using (15), we determine $\partial T(x, \tau)/\partial x$:

$$\frac{\partial T(x, \tau)}{\partial x} = -k + T_a \sqrt{\frac{\left[\frac{dR(x)}{dx}\right]^2 + \left[\frac{dS(x)}{dx}\right]^2}{V^2 + U^2}} \sin[\omega\tau + \varphi(x) + \psi(x)], \tag{21}$$

where

$$\psi(x) = \operatorname{arc\,tg} \left[\frac{R(x)\frac{dS(x)}{dx} - S(x)\frac{dR(x)}{dx}}{R(x)\frac{dR(x)}{dx} + S(x)\frac{dS(x)}{dx}} \right]; \tag{22}$$

$$\frac{dR(x)}{dx} = \frac{z}{\delta} P(x); \tag{23}$$

$$\frac{dS(x)}{dx} = -\frac{z}{\delta} W(x); \tag{24}$$

$$P(x) = \operatorname{sh}\left(z\frac{\delta - x}{\delta}\right) \sin\left(z\frac{\delta - x}{\delta}\right) - \operatorname{ch}\left(z\frac{\delta - x}{\delta}\right) \cos\left(z\frac{\delta - x}{\delta}\right) + \frac{2z}{\mathrm{Bi}} \operatorname{ch}\left(z\frac{\delta - x}{\delta}\right) \sin\left(z\frac{\delta - x}{\delta}\right); \tag{25}$$

$$W(x)=\operatorname{sh}\left(z\frac{\delta-x}{\delta}\right)\sin\left(z\frac{\delta-x}{\delta}\right)+\operatorname{ch}\left(z\frac{\delta-x}{\delta}\right)\cos\left(z\frac{\delta-x}{\delta}\right)+\frac{2z}{\mathrm{Bi}}\operatorname{sh}\left(z\frac{\delta-x}{\delta}\right)\cos\left(z\frac{\delta-x}{\delta}\right). \tag{26}$$

Substituting (23) and (24) into (21) and then (21) into (20):

$$q_r=k\lambda-T_a\frac{\lambda z}{\delta}\sqrt{\frac{P^2(0)+W^2(0)}{V^2+U^2}}\sin[\omega\tau+\varphi(0)+\psi(0)]. \tag{27}$$

Analyzing Eq. (19), we note that the average value of the specific thermal flux found from the experimental data is obtained for $\sin(\omega\tau+\rho)=0$. Then

$$q_{e_{av}}=\lambda k. \tag{28}$$

It is precisely this mean thermal flux which passes from the casting to the coolant in actual fact.

From Eq. (27) with $\sin[\omega\tau+\varphi(0)+\psi(0)]=0$ we obtain

$$q_{r_{av}}=\lambda k. \tag{29}$$

In determining the thermal operating conditions of molds used for the continuous casting of metals, it is very important to know the maximum thermal fluxes.

The maximum value of the specific thermal flux determined by the maximum temperature drop at the thermocouple junctions may be found from Eq. (19) with $\sin(\omega\tau+\rho)=1$:

$$q_{e_{max}}=\lambda k+\frac{\lambda T_a}{x_2-x_1}\sqrt{\frac{[R(x_1)-R(x_2)]^2+[S(x_1)-S(x_2)]^2}{V^2+U^2}}. \tag{30}$$

The maximum value of the real thermal flux is found from Eq. (27) with $\sin[\omega\tau+\varphi(0)+\psi(0)]=-1$:

$$q_{r_{max}}=k\lambda+T_a\frac{\lambda z}{\delta}\sqrt{\frac{P^2(0)+W^2(0)}{V^2+U^2}}. \tag{31}$$

Dividing (31) termwise by (30), we obtain

$$\frac{q_{r_{max}}}{q_{e_{max}}}=\frac{1+m_1\dfrac{T_a}{T_0-T_B}}{1+m_2\dfrac{T_a}{T_0-T_B}}, \tag{32}$$

where

$$m_1=z\left(1+\frac{1}{\mathrm{Bi}}\right)\sqrt{\frac{P^2(0)+W^2(0)}{V^2+U^2}}; \tag{33}$$

$$m_2=\frac{\delta}{x_2-x_1}\left(1+\frac{1}{\mathrm{Bi}}\right)\sqrt{\frac{[R(x_1)-R(x_2)]^2-[S(x_1)-S(x_2)]^2}{V^2+U^2}}. \tag{34}$$

Using Eq. (15), we express the mean and peak temperatures on the surface of the mold wall turned toward the casting T_0 and T_a in terms of T_0' and T_a', the mean and peak temperatures on the surface of the wall turned toward the coolant. Then:

$$\frac{T_a}{T_0-T_B}=\frac{T_a'}{T_0'-T_B}\cdot\frac{\sqrt{V^2+U^2}}{\sqrt{2}\,z\left(1+\dfrac{1}{\mathrm{Bi}}\right)}. \tag{35}$$

Substituting (35) into (32), we obtain

$$q_{r_{max}}=q_{e_{max}}\frac{1+k_1\theta}{1+k_2\theta}, \tag{36}$$

where

$$k_1=\frac{1}{\sqrt{2}}\sqrt{P^2(0)+W^2(0)}; \tag{37}$$

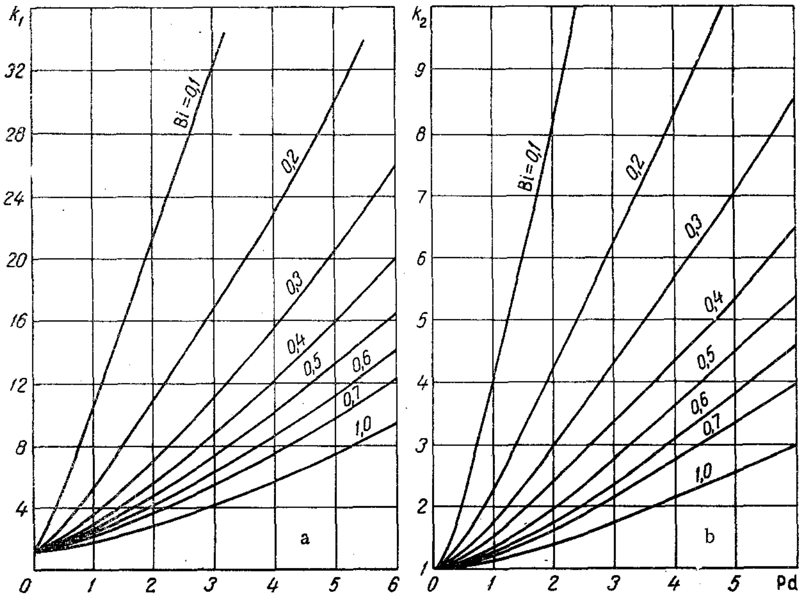

Fig. 3. Relation between the Bi and Pd criteria and the coefficients k_1 (a) and k_2 (b).

$$k_2 = \frac{\delta}{x_2 - x_1} \frac{1}{\sqrt{2z}} \sqrt{[R(x_1) - R(x_2)]^2 + [S(x_1) - S(x_2)]^2}\,; \tag{38}$$

$$\theta = \frac{T_a'}{T_0' - T_\text{в}}. \tag{39}$$

Using Eqs. (19) and (27), we may also determine the minimum value of the specific thermal flux calculated from the experimental data for $\sin(\omega\tau + \rho) = -1$, and the real values for $\sin[\omega\tau + \varphi(0) + \psi(0)] = 1$:

$$\frac{q_{r_{min}}}{q_{e_{min}}} = \frac{1 - k_1\theta}{1 - k_2\theta}. \tag{40}$$

Using (28) and (29) we may derive an equation for θ:

$$\frac{1}{2}(q_{r_{max}} + q_{r_{min}}) = \frac{1}{2}(q_{e_{max}} + q_{e_{min}}). \tag{41}$$

Substituting (36) and (40) into (41) we obtain

$$\theta = \frac{\varepsilon - 1}{k_2(\varepsilon + 1)}, \tag{42}$$

where

$$\varepsilon = \frac{q_{e_{max}}}{q_{e_{min}}}. \tag{43}$$

Thus Eq. (36) may be used in order to determine the maximum values of the true thermal fluxes in the mold.

In order to ease the practical use of Eq. (36), Fig. 3a, b gives the coefficients k_1 and k_2 in the form of curves plotted for various Bi and Pd criteria.

In plotting the curve of k_2 from (38), it was assumed that $x_2 = \delta$ and $x_1 = 0.2\delta$, i.e., that the cold junction of the thermocouple lay on the surface of the wall washed by the coolant and the hot junction at a distance of 0.2δ from the surface of the wall turned toward the casting. For a mold wall thickness of 12 mm the thermocouple junctions were arranged in approximately the same way.

The method proposed for determining the thermal fluxes in the plane walls of the mold is as follows.

A differential thermocouple is placed in the wall of the mold and connected to an automatically-recording potentiometer. Using the recording of the electromotive force developed by the thermocouple, we determine $q_{e_{max}}$, $q_{e_{av}}$, and $\varepsilon = q_{e_{max}}/q_{e_{min}}$ by means of Eq. (1).

Apart from the differential thermocouple, another thermocouple is placed on the surface of the wall on the side of the coolant and connected to another automatic recorder. The readings of this latter give T_0' and T_a'.

Knowing the temperature of the coolant T_B, Eq. (39) may be used to determine θ. Then we find k_2 from (42), and then the Bi criterion from Fig. 3b. The angular frequency for finding the Pd criterion is determined from the diagrams recorded by the potentiometer.

Finally, using Fig. 3a, we may determine k_1, and then by Eq. (36) $q_{r_{max}}$. As already indicated, $q_{e_{av}} = q_{r_{av}}$.

NOTATION

ΔT	is the temperature difference between the differential thermocouple junctions;
Δx	is the distance between the differential thermocouple junctions;
λ	is the thermal conductivity of the mold wall material;
T_0, T_0'	are the mean temperatures of the wall surfaces facing the casting and coolant, respectively;
T_a, T_a'	are the peak values of the corresponding temperatures;
ω	is the angular frequency of temperature variation;
δ	is the mold wall thickness;
T_B	is the temperature of coolant;
α	is the heat-transfer coefficient between wall and coolant;
τ	is the time;
Bi	is the Biot number;
Pd	is the Predvoditelev number.

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